$$(y'^2-b^2)x^2+2x'x-(a^2y'^2+b^2x'^2)+c=0\text{.........}(4),$$

and the condition that (4) has equal roots, or that the hyperbola touches the X-axis is given by $y'^2(a^2b^2-a^2y'^2-b^2x'^2)=c(b^2-y'^2)$ ....(5); and, in a similar way that the curve touches the Y-axis, $x'^2(a^2b^2-a^2y'^2-b^2x'^2)=c(a^2-x'^2)$ ....(6). (5)÷(6) gives after reduction, $a^2y'^2-b^2x'^2=0$..........(7), showing that (x', y') is on an equi-conjugate axis of the ellipse.

Also solved by G. W. Greenwood, A. H. Holmes, W. W. Landis, J. Scheffer, and G. B. M. Zerr.

GROUP THEORY.

12. Proposed by GEORGE H. HALLETT, Ph. D., Assistant Professor of Mathematics, The University of Pennsylvania.

Given $U_1=\alpha'$, $V_1=\beta'$, and the recursion formulae $U_y=\alpha' V_{y-1}+\alpha'' U_{y-1}$, $V_y=\beta' V_{y-1}+\beta'' U_{y-1}$. Find expressions for U_y, V_y in terms of the coefficients α', α'', β', β''.

Solution by PROFESSOR JAMES BYRNIE SHAW, The James Milliken University, Decatur, Ill.

By eliminating V we find that U_n is the coefficient of x^n in the expansion of

$$\frac{\alpha' x}{1-(\alpha''+\beta')x+(\alpha''\beta'-\alpha'\beta'')x^2}.$$

Likewise we find that V_n is the coefficient of x^n in the expansion of

$$\frac{1-\alpha'' x}{1-(\alpha''+\beta')x+(\alpha''\beta'-\alpha'\beta'')x^2}.$$

We may state the result as follows: Let $\cos\theta=\frac{1}{2}.\frac{\alpha''+\beta'}{T}$ where $T^2=\begin{vmatrix} \alpha'' & \alpha' \\ \beta'' & \beta' \end{vmatrix}$.

Then $V_n=\alpha'.T^{n-1}.\frac{\sin n\theta}{\sin\theta}$, and $V_n=T^n.\frac{\sin(n+1)\theta}{\sin\theta}-\alpha'' T^{n-1}\frac{\sin n\theta}{\sin\theta}$. These latter forms are easily verified by mathematical induction. The well-known formulae for $\frac{\sin n\theta}{\sin\theta}$ give U_n and V_n in terms of the coefficients directly, and free from irrationalities.

13. Proposed by O. E. GLENN, Ph. D., Springfield, Mo.

The order of the linear homogeneous group in n letters is $(p^n-1)(p^n-p)$ ....(p^n-p^{n-1}). Two proofs are given in Burnside's *Finite Groups*. Give other proofs.

Solution by the PROPOSER.

The linear homogeneous group is known to be equivalent to the group of isomorphisms of the abelian group $H_{p^n}=[P_1, P_2 \text{.........} P_n]$ of type [1 1 1.........],

order p^n. Let $h_{i_1}\ h_{i_2}$ .... represent the subgroups of H, of order p, and $J_{ik}=\begin{pmatrix}P_1 & P_2 & \cdots\cdots P_n \\ h_{i_1} & h_{i_2} & \cdots\cdots h_{i_n}\end{pmatrix}$ the isomorphism of H gotten by replacing P_j by any operation (order p) in h_{ij} ($j=1\ 2$ .... m), say the new generators from the kth set of all of the possible sets which might be chosen from h_{i_1}, h_{i_2} .... h_{i_n}. The number of values of k is obviously equal to $\Phi(p)^n=(p-1)^n$. To determine the number of choices *of this set* of subgroups (number of values of i) suppose that a of a set of n generators have been selected. The remaining $n-a$ operations must be selected outside the subgroup H_{p^a} generated by the first a, and thus there remain

$$\frac{p^n-1}{p-1}-\frac{p^a-1}{p-1}=\frac{p^a(p^{n-a}-1)}{p-1}$$

subgroups h_{ij} from which to select the remaining $n-a$. Thus the product of the number of values of k and the number of values of i is

$$h=(p-1)^n\prod_{a=0}^{n-1}\frac{p^a(p^{n-a}-1)}{p-1}=(p^n-1)(p^n-p)(p^n-p^2)\cdots\cdots(p^n-p^{n-1})$$

which is the number of choices of new generators of H, or the order of its automorph.

MECHANICS.

186. Proposed by R. D. CARMICHAEL, Hartselle, Alabama.

A point P keeps at uniform distance from and moves with uniform angular velocity around a point Q which is in harmonic motion, making one revolution while Q swings to and fro. If P is in the line of the path of Q and on the same side of the center of that path with Q when Q is at the extremity of the path, what is the locus of P?

Solution by the PROPOSER.

Take the origin at the center of the path of Q, and let a=half the length of that path. Let $PQ=b$, and let θ=the angle of PQ with the path of Q at any time. Then, it is easily shown that $x=(a+b)\cos\theta$, $y=b\sin\theta$, the equations of an ellipse whose axes are $a+b$ and b.

Also solved by G. W. Greenwood, and G. B. M. Zerr.